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Global Symmetries in the Antifield–Formalism

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Abstract

In this paper, two things are done. (i) First, it is shown that any global symmetry of a gauge-invariant theory can be extended to the ghosts and the antifields so as to leave invariant the solution of the master-equation (before gauge fixing). (ii) Second, it is proved that the incorporation of the rigid symmetries to the solution of the master-equation through the introduction of a constant ghost for each global symmetry can be obstructed already at the classical level whenever the theory possesses higher order conservation laws. Explicit examples are given.

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1 Introduction

This letter is devoted to the implementation of global ($\equiv$ rigid) symmetries in the antifield-formalism.

Consider a gauge-invariant theory with gauge-invariant local action

$$I = \int \mathrm{d}^n x \, \mathcal{L}\left([\phi], x\right) \tag{1}$$

and gauge-symmetries

$$\delta_\eta \, \phi^i = \sum_{k=0}^{t} r^{i\,\mu_1 \cdots \mu_k}_{\alpha} \left([\phi], x\right) \partial_{\mu_1 \cdots \mu_k} \eta^\alpha \tag{2}$$

where η^α are arbitrary space-time functions and where the argument $[\phi]$ indicates dependence on the fields and on their derivatives $\partial_\mu \phi^i, \ldots, \partial_{\mu_1 \cdots \mu_r} \phi^i$ up to some finite (but arbitrary) order $r < \infty$. It has been established in [1] that under quite general regularity conditions on the Lagrangian, the gauge-symmetries and the reducibility functions (if any), there exists a local solution of the classical master equation introduced by Zinn-Justin as well as Batalin and Vilkovisky [2, 3, 4],

$$(S_0, S_0) = 0, \quad S_0 = I \; + \; \text{“more”} \;\;, \tag{3}$$

where “more” is the integral of a local function that involves at least one ghost and one antifield in each of its field monomials. The proof given in [1] follows the lines of homological perturbation theory [5] and adapts to local functionals the proofs given in [6, 7, 8] for irreducible gauge-theories and in [9] for reducible ones (for a general discussion of the homological tools underlying the antifield formalism, see [10]).

Besides the gauge-symmetries (2), the theory may possess also global symmetries (e.g. Poincaré-symmetry, rigid supersymmetry, global internal symmetries etc.),

$$\delta_\epsilon \, \phi^i = t^i_A \left([\phi], x\right) \epsilon^A \equiv \left(\delta_A \phi^i\right) \epsilon^A \;, \tag{4}$$

where ϵ^A are constant parameters. The action (1) is invariant under (4),

$$\delta_\epsilon \, I \; = \; 0 \, . \tag{5}$$

The rigid symmetry (4) may or may not be linearly realized, i.e. the functions t^i_A need not be linear (homogeneous) in the fields and their derivatives. Furthermore, the commutator of a rigid symmetry with a gauge-symmetry may involve both a non-zero gauge-symmetry and on-shell trivial gauge-symmetries, while the rigid symmetries may close only up to a gauge-symmetry and on-shell trivial transformations. Schematically,

$$[\delta_A, \delta_\alpha] \;\phi^i \;\; = \;\; \mu^\beta_{A\alpha} \, R^i_\beta \; + \; \mu^{ij}_{A\alpha} \, \frac{\delta I}{\delta \phi^j} \;, \tag{6}$$

$$[\delta_A, \delta_B]\ \phi^i \;=\; \lambda_{BA}^C\, \delta_C\, \phi^i \;+\; \lambda_{BA}^\alpha\, R_\alpha^i \;+\; \lambda_{BA}^{ij}\, \frac{\delta I}{\delta \phi^j}\,, \tag{7}$$

where we have set $\delta_\eta\, \phi^i \equiv \eta^\alpha R_\alpha^i \equiv \eta^\alpha \delta_\alpha \phi^i$ and where we have used temporarily DeWitt's condensed notations. In (7), the λ_{AB}^C are the *structure constants* of the (graded) rigid symmetry Lie algebra.

The first point investigated in this letter is whether the solution S_0 of the master equation (3) remains invariant under the global symmetry (4). More precisely: Is it possible to modify the transformation law (4) by ghost dependent contributions (so that (4) is unchanged if the ghosts are set equal to zero) and define the transformation rules for the ghosts and the antifields in such a way that $\delta_\epsilon\, S_0 = 0$? The answer to this question turns out to be always affirmative and, furthermore, the global symmetry is canonically generated in the antibracked by a *local* generator S_A of ghost-number minus one

$$S_A \;=\; \int \mathrm{d}^n x \left(\phi_i^*\, t_A^i \;+\; \text{“more”}\right) \tag{8}$$

such that

$$\delta_\epsilon\, z^\Delta \;=\; \left(z^\Delta, S_A\right) \epsilon^A \tag{9}$$

for all variables z^Δ, including the ghosts and the antifields, and

$$(S_0, S_A)\, \epsilon^A \;=\; \delta_\epsilon\, S_0 \;=\; 0\,. \tag{10}$$

In order to analyse whether a global symmetry is quantum-mechanically anomalous, and if not, how it gets renormalized, it is convenient to consider an extended generating functional S that incorporates both the gauge-symmetries and the global symmetry,

$$S \;=\; S_0 \;+\; S_A\, \xi^A \;+ \tfrac{1}{2}\, S_{AB}\, \xi^B\, \xi^A \;+\; O\left(\xi^3\right)\,, \tag{11}$$

where ξ^A are constant ghosts associated with the global symmetry, such that

$$(S, S) \;+\; \frac{\partial^R S}{\partial \xi^C}\, \lambda_{BA}^C\, \xi^A\, \xi^B\ (-1)^{\varepsilon_B} \;=\; 0 \tag{12}$$

(“extended master equation”). Here, ε_A is the parity of the global symmetry δ_A and the λ_{AB}^C are the structure constants appearing in (7). Equation (12) guarantees the existence of a nilpotent antiderivation D ($D^2 = 0$) that encodes both the gauge and global symmetries and is defined on any function(al) X of the fields, antifields and constant ghosts by

$$D\, X \equiv (X, S) + \tfrac{1}{2}\, (-1)^{\varepsilon_B}\, \frac{\partial^R X}{\partial \xi^C}\, \lambda_{BA}^C\, \xi^A\, \xi^B\,. \tag{13}$$

Such an extended generator S or antiderivation D have been constructed by various authors mostly (but not only) in the context of globally supersymmetric

theories [3, 12, 13]. It has also been analysed recently in [14] in connection with equivariant cohomology.

Now, the second question addressed in this letter is whether the existence of a *local* solution of the extended master equation (12) starting like (11) is always guaranteed. It is shown that the answer to this question may be negative whenever the theory has higher order non-trivial conservation laws,

$$\partial_{\mu_1} j^{[\mu_1 \cdots \mu_k]}([\phi], x) \approx 0 \ , \quad j^{[\mu_1 \cdots \mu_k]}([\phi], x) \not\approx \partial_{\mu_0} \omega^{[\mu_0 \cdots \mu_k]}([\phi], x) \quad (k \geq 2), \tag{14}$$

where $\approx$ denotes weak ($\equiv$ on-shell) equality and $[\mu_1 \cdots \mu_k]$ complete antisymmetrization. Thus, even though any global symmetry can be extended to the space of the ghosts and antifields in such a way that it leaves the solution S_0 of the "restricted master equation" (3) invariant, it may be impossible to complete $S_0 + S_A \xi^A$ to a local solution S of equation (12). We provide explicit examples where obstructions arise. We also give conditions for the expansion (11) to be unobstructed so that the extended formalism based on (12) exists. These conditions are met, for example, in Yang-Mills gauge-theories, gravity as well as Super-Yang-Mills models. [As usual in the context of the antifield formalism, we say that a functional is local if each term in its expansion according to the antighost-number is the integral of a function of the fields, the ghosts, the antifields and a finite number of their derivatives ("local function").]

2 Implementation of Global Symmetries in the Antifield Formalism

Our first task is to investigate whether the global symmetry (4) of the classical action I is also a symmetry of the solution S_0 of the master equation (3). Since S_0 involves more variables than I does, what we really mean by this question is whether one can modify the global symmetry (4) by ghost dependent terms (invisible when the ghosts are set equal to zero) and define appropriate transformation rules for the ghosts and the antifields in such a way that $\delta_\epsilon S_0 = 0$. It cannot be stressed enough that any "proof" of invariance of S_0 under a given global symmetry that does not indicate at the same time how to transform the ghosts and antifields is incomplete and meaningless.

We claim that the answer to this first question is always positive and that, moreover, the symmetry in the extended space is canonically generated in the antibracket, as in equation (9). To prove this statement, let us construct directly the canonical generator S_A.

In order to reproduce (4) through (9), it is necessary that S_A starts like in equation (8), where "more" contains at least one local ghost of the gauge-symmetry and accordingly has antighost-number greater than one. (For more information on our grading conventions and on the Koszul-Tate differential in the

antifield formalism used below, see [9, 10].) Now, the equation $\delta_\epsilon\, I = 0$ is equivalent, in terms of the Koszul-Tate differential δ, to the condition $\int \mathrm{d}^n x\ \delta\left(\phi^*_i\ t^i_A\right) = 0$, i.e., $\delta\left(\phi^*_i\ t^i_A\right)\ +\ \partial_\mu j^\mu = 0$ for some j^μ. This means that the first term $\phi^*_i\ t^i_A$ of S_A defines a cocycle of the cohomology $H_1\left(\delta|d\right)$, which is non-trivial because we assume the given rigid symmetry to be itself non-trivial. By using Theorem 6.1 of reference [15] on the isomorphism between $H_k\left(\delta|d\right)$ and the BRST-cohomology modulo d at negative ghost number $-k$,

$$H^{-k}\left(s|d\right)\ \simeq\ H_k\left(\delta|d\right) \quad (k > 0)\ , \tag{15}$$

it is possible to infer the existence of a local "BRST-invariant extension" S_A of $\int \mathrm{d}^n x\left(\phi^*_i\ t^i_A\right)$ with the required property

$$s\, S_A\ =\ 0\ . \tag{16}$$

Here, s is the BRST differential associated with the *gauge*-symmetry, $sA = (A, S_0)$. The proof of (15) and of the resulting existence of S_A uses the acyclicity of δ in the space of local functionals containing at least one local ghost of the gauge-symmetry and one antifield in each term [1].

The equation (16) can be rewritten as

$$(S_A, S_0)\ =\ 0 \tag{17}$$

since the BRST-transformation s is canonically generated by S_0. If one defines the transformation rules for all the variables according to (9), one sees that equation (17), which expresses the BRST-invariance of S_A, can be read backwards and expresses also the invariance of S_0 under the symmetry generated by S_A. This answers positively the first question raised above. Note that since S_A is a local functional, the transformation rules of all the variables z^Δ, $\delta_\epsilon z^\Delta = \left(z^\Delta, S_A\right)\epsilon^A$, are local functions.

It is well known that the solution S_0 of the master equation (3) carries some ambiguity. In the recursive construction of S_0, one has the possibility to add at each stage an arbitrary δ-exact term. One could choose these higher order terms involving the ghosts and the antifields in a manner that could conflict with some preconceived idea of manifest invariance under the global symmetry of the theory. What our result indicates is that this does not matter. One can choose the higher order terms in a way which is not manifestly invariant since it is always possible to extend the transformation laws in the space of the fields, the ghosts and the antifields so that S_0 is strictly invariant, no matter how one has fixed the ambiguity in S_0.

By using (6), one verifies easily that in S_A,

$$\begin{aligned} S_A\ &=\ S_{A,(1)} + S_{A,(2)}\ + \cdots\ , \\ S_{A,(1)}\ &=\ \int \mathrm{d}^n x\ \phi^*_i\ t^i_A\ , \end{aligned}$$

the term of antighost-number two is given by

$$S_{A,(2)} \sim C^*_\beta \, \mu^\beta_{A\alpha} \, C^\alpha \; + \; \tfrac{1}{2} \, \phi^*_i \, \phi^*_j \, \mu^{ij}_{A\alpha} \, C^\alpha \; , \tag{18}$$

where we use again the condensed notation (and '$\sim$' to indicate that parity dependent phase factors are suppressed). The first term on the right hand side of (18) determines the transformation rule for the local ghosts to first order; the second term modifies the transformation rule of the fields ϕ^i by the term $\phi^*_j \, \mu^{ij}_{A\alpha} \, C^\alpha$ and, according to (6), arises only if the commutator of a rigid symmetry with a gauge-symmetry involves an on-shell trivial symmetry.

Since the S_A are BRST-closed, their antibracket is also BRST-closed. By using (7), one finds that $(S_A, S_B) - \lambda^C_{AB} \, S_C$ is a BRST cocycle of ghost number minus one whose component of antighost-number one is δ-exact in the space of local functionals,

$$\begin{aligned} (S_A, S_B) - \lambda^C_{AB} \, S_C \quad \sim \quad & \delta \, \left(C^*_\alpha \, \lambda^\alpha_{AB} + \tfrac{1}{2} \, \phi^*_i \, \phi^*_j \, \lambda^{ij}_{AB} \right) \\ & \qquad + \text{terms of higher antighost-number.} \end{aligned}$$

Thus, according to the results of [15] and [1], $(S_A, S_B) - \lambda^C_{AB} S_C$ is actually s-exact, i.e. there exists a local functional S_{AB} of ghost-number -2 such that

$$(S_A, S_B) - \lambda^C_{AB} S_C - (-1)^{\varepsilon_A} \, (S_0, S_{AB}) = 0. \tag{19}$$

If one redefines S_A as $S_A \longrightarrow S_A + (S_0, K_A)$, where K_A has ghost-number -2, which is tantamount to redefining the global symmetry by gauge-symmetries and on-shell trivial symmetries, one finds that S_{AB} transforms as

$$S_{AB} \longrightarrow S_{AB} - (-1)^{\varepsilon_A} (\lambda^C_{AB} K_C + M_{AB} - (-1)^{\varepsilon_A \varepsilon_B} M_{BA})$$

with $M_{AB} = (K_A, S_B) - \frac{1}{2}(K_A, (S_0, K_B))$.

The above derivation mimics exactly the treatment of [16] of rigid symmetries in the Hamiltonian version of BRST theory, where the term (S_{AB}, S_0) was called the "BRST extension" of the Lie algebra defined by λ^C_{AB}. In fact, it is possible to apply the antifield approach also to the Hamiltonian formalism [17, 18, 19]. Doing this in the present context, one finds for S_A a formula analogous to the formula connecting S_0 and the canonical BRST generator Ω [17, 18], i.e.

$$S_A \sim \int \mathrm{dt} \left(q^*_i \left[q^i, Q_A \right] + p^{*i} \left[p_i, Q_A \right] + \lambda^a \left[\varrho_a, Q_A \right] + \eta^*_a \left[\eta^a, Q_A \right] \right), \tag{20}$$

where Q_A are the canonical generators of the rigid symmetry in the extended phase space.

3 Extended Master Equation

We now turn to the question whether a local solution to the master equation (12) incorporating the global symmetries is guaranteed to exist. As we shall show, the answer may be negative.

To analyse the question, we expand the searched-for S in powers of the global ghosts ξ^A as in [14] (see equation (11)). The coefficients S_A, S_{AB}, S_{ABC} ... should be local functionals of decreasing ghost-number -1, -2, -3, ... With S_A and S_{AB} constructed as above, the extended master equation holds up to order ξ^2 included, i.e.

$$\left(S^{(2)}, S^{(2)}\right) + \frac{\partial^R S^{(2)}}{\partial \xi^C} \lambda^C_{BA}\, \xi^A\, \xi^B\, (-1)^{\varepsilon_B} = O\left(\xi^3\right)$$

where $S^{(2)} = S_0 + S_A \xi^A + \frac{1}{2} S_{AB} \xi^B \xi^A$. Therefore, let us proceed recursively. Assume that one has constructed S up to order ξ^k, $S^{(k)} = S_0 + S_A\, \xi^A + \cdots + \frac{1}{k!} S_{A_1 \cdots A_k}\, \xi^{A_k} \ldots \xi^{A_1}$, so that the extended master equation holds up to order k included, and let us try to determine the term of order $k+1$ in S so that $S^{(k+1)}$ solves the extended master equation (12) up to order $k+1$ included.

The equation for $S_{A_1 \cdots A_{k+1}}$ that follows from (12) takes the form

$$\left(S_{A_1 \cdots A_{k+1}}, S_0\right) \; = \; R_{A_1 \cdots A_{k+1}} \; , \tag{21}$$

where the local functional $R_{A_1 \cdots A_{k+1}}$ has ghost-number $-k$ and is built out of the already constructed $S_{A_1 \cdots A_j}$. Thus, in order for $S_{A_1 \cdots A_{k+1}}$ to exist, $R_{A_1 \cdots A_{k+1}}$ should be s-exact in the space of local functionals. By using the Jacobi-identity for the antibracket, one easily checks that $R_{A_1 \cdots A_{k+1}}$ is s-closed. The proof follows the standard pattern of homological perturbation theory [5, 10] and will not be repeated here. Accordingly, a sufficient condition for S_{ABC}, $S_{ABCD}, \ldots, S_{A_1 \cdots A_{k+1}}, \ldots$ to exist is that the BRST cohomological groups in the space of local functionals, denoted by $H^j\left(s|d\right)$, vanish[1] for $j \leq -2$.

Now, as we have already recalled, one has $H^{-k}\left(s|d\right) \simeq H_k\left(\delta|d\right)$, where δ is the Koszul-Tate differential associated with the equations of motion. Furthermore, $H_k\left(\delta|d\right)$ is itself isomorphic to the characteristic cohomology $H^{n-k}_{char}\left(d\right)$ of Vinogradov [20] and Bryant and Griffiths [21], which describes the higher-order conservation laws (14) [15]. Thus, obstructions to the existence of the higher-order terms $S_{A_1 \cdots A_{k+1}}$, i.e. to the existence of a local functional solution of the extended master equation (12), may arise whenever there exist non-trivial higher-order conservation laws in the theory.

[1]Strictly speaking, the cohomology $H^j\left(s\right)$ of s in the space of local functionals is not exactly the same as the cohomology $H^j\left(s|d\right)$ in the space of local volume forms because the surface terms that one drops in calculating $H^j\left(s|d\right)$ may be non zero. Thus, a cocycle of $H^j\left(s|d\right)$ does not yield necessarily a cocycle of $H^j\left(s\right)$ upon integration. However, when discussing locality, it is really $H^j\left(s|d\right)$ that is relevant. Hence, we deal exclusively from now on with $H^j\left(s|d\right)$.

Of course, the presence of higher-order conservation laws does not necessarily lead to obstructions. It could happen that $R_{A_1 \cdots A_{k+1}}$ in (21) is always in the trivial class of $H^{-k}(s|d)$. This would be for instance the case if $R_{A_1 \cdots A_{k+1}}$ contained no term of antighost-number k. Our main result, however, is that there exist global symmetries for which the obstructions are effectively present[2]. We shall establish this point by means of an explicit example.

A theory with a non-vanishing second homology group $H_2(\delta|d)$ is the free Maxwell-theory without sources. In that case, $H_2(\delta|d)$ is one-dimensional (except in two dimensions). One may take as representative of the non-trivial cohomology class of $H_2(\delta|d)$ the antifield C^* associated with the ghost field C ($\delta C^* = -\partial_\mu A^{*\mu}$). This class corresponds to the conservation law $\partial_\mu F^{\mu\nu} \approx 0$.

The Maxwell action in Minkowski-space is invariant, among other symmetries, under translations,

$$\delta_a \, A_\mu \;=\; a^\nu \partial_\nu A_\mu \;, \tag{22}$$

and under the following x-dependent shifts in the fields,

$$\delta_b \, A_\mu \;=\; b_{\mu\nu} \, x^\nu \;, \qquad\qquad b_{\mu\nu} = -b_{\nu\mu} \tag{23}$$

where a^μ and $b_{\mu\nu}$ are constant parameters. [The symmetric part of $b_{\mu\nu}$ defines a gauge-symmetry and is excluded from the discussion for that reason.] The symmetries (22) and (23) commute up to a gauge-transformation. Thus, $\lambda^C_{AB} = 0$. We shall show that the construction of a solution of the extended master equation (12) incorporating (22) and (23) is obstructed.

The solution of the "restricted" master equation (3) reads

$$S_0 = \int \mathrm{d}^n x \left(-\tfrac{1}{4}\, F_{\mu\nu}\, F^{\mu\nu} + A^{*\mu}\, \partial_\mu C\right) \;, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \;. \tag{24}$$

The BRST-invariant term $S_1 \equiv \xi^A S_A$ generating the global symmetries (22) and (23) is

$$S_1 = \int \mathrm{d}^n x \left[A^{*\mu} \left(\xi^\nu \, \partial_\nu A_\mu + \xi_{\mu\nu}\, x^\nu\right) \;-\; C^* \, \xi^\mu \partial_\mu C\right] \;, \tag{25}$$

where ξ^ν and $\xi_{\mu\nu}$ are constant anticommuting ghosts associated with the translations (22) and with the symmetry (23) respectively. The generator (25) determines the transformation properties of the antifields and the ghosts under the given rigid symmetries. By computing their antibracket with S_1, one finds that these do not transform under (23), while they behave as

$$\delta_a z^\Delta = a^\mu \partial_\mu z^\Delta \qquad \forall \; z^\Delta \in \{C,\; A^{*\mu},\; C^*\}$$

under translations, as expected.

[2] Note that the term of antighost-number k of $R_{A_1 \cdots A_{k+1}}$ does not involve the local ghosts associated with the gauge-symmetries, so the argument of [1] on the vanishing of $H_i(\delta|d)$ ($i \geq 1$) when local ghosts are present does not apply.

Although one could add to S_1 a BRST-exact functional, we shall not do it here. This is because we want to stick to the original form (22) and (23) of the rigid symmetries, without modifying them by the addition of gauge-transformations or on-shell trivial symmetries. Also, we want to maintain the quadratic character of S_1.

With $\lambda^C_{AB} = 0$, the extended master equation reads simply $(S, S) = 0$. We are looking for a solution of that equation of the form $S = S_0 + S_1 + S_2 + \cdots$, where S_k has degree k in the constant ghosts ξ^ν and $\xi_{\mu\nu}$. The extended master equation requires $(S_1, S_1) + 2\ (S_0, S_2) = 0$ for some S_2. This implies

$$S_2 = \int \mathrm{d}^n x\, (C^*\ x^\mu\ \xi_{\mu\nu}\ \xi^\nu) \tag{26}$$

up to a BRST-closed term, which reflects the fact that the commutator of the global symmetries (22) and (23) is a gauge-transformation with parameter $a^\mu b_{\mu\nu} x^\nu$.

It is for the next term S_3 that one meets the obstruction: one finds indeed

$$(S_1, S_2) = \xi^\mu\ \xi^\nu\ \xi_{\mu\nu} \int \mathrm{d}^n x\, C^* \tag{27}$$

the integrand of which is non-trivial in $H_2\,(\delta|d)$. Therefore, there is no S_3 such that $(S_1, S_2) + (S_0, S_3) = 0$ holds, i.e. a local solution of the master equation (12) encoding the above rigid symmetries *simply does not exist.* Note that the ambiguity in S_2 [$S_2 \longrightarrow S_2 + M_2$ where M_2 is BRST-closed and of antighost-number at least two, i.e. $S_2 \longrightarrow S_2 + f(\xi) \int \mathrm{d}^n x\, C^* + (S_0, K_2)$] does not allow for the removal of the obstruction (27).

We note however that one can indeed remove the obstruction by further extending the formalism and the master equation (12). To that end we introduce another constant ghost, Q, associated with the global reducibility identity on the gauge symmetry responsible for the non-vanishing of $H_2(s|d)$ [15]. We assign to it ghost number 2 and even Grassmann parity. We also introduce a constant anti'field' Q^* conjugate to Q. Then, with S_0, S_1 and S_2 as above,

$$S = S_0 + S_1 + S_2 + \int \mathrm{d}^n x\, C^* Q - Q^*\, \xi^\mu \xi^\nu \xi_{\mu\nu}$$

solves an extended master equation in the standard form, $(S, S) = 0$, where the antibracket now involves also (ordinary) derivatives w.r.t. Q and Q^* (analogously one can always cast (12) in the standard form by introducing ξ^*_A).

The example can be generalized to free 2-form gauge-fields, for which $H_2\,(\delta|d) = 0$ but $H_3\,(\delta|d) \neq 0$ [22]. The symmetries $B_{\mu\nu} \longrightarrow B_{\mu\nu} + a^\varrho \partial_\varrho B_{\mu\nu} + b_{\mu\nu\varrho} x^\varrho$ (with $b_{\mu\nu\varrho}$ completely antisymmetric) can be incorporated in the master equation (12) up to order S_3 included ($H_2\,(\delta|d) = 0$) but get obstructed at the next level. Again, one can remove the obstruction by introducing a constant ghost with ghost number 3 and odd Grassman parity. One finds similar results for free p-form gauge-fields with $p > 2$.

4 Conclusions

In this letter, we have proved that any global symmetry of a gauge-invariant theory can always be extended to the antifields and the ghosts so as to be a symmetry of the restricted (usual) master equation (3). We emphasize that this holds for the solution of the master equation *before* gauge fixing and that, in general, our result does not imply an analogous property for the gauge fixed action (see [12] for a discussion of this problem). We have then discussed the incorporation of the rigid symmetries in the extended formalism and have shown that one cannot take for granted the existence of a local solution of the extended master equation (12) in which the rigid symmetries are included with constant ghosts. Indeed, one can meet obstructions and we have provided explicit examples for that.

The possible obstructions are given by the cohomological groups $H_i\,(\delta|d)$, $(i \geq 2)$ of the characteristic cohomology. Thus, when $H_i\,(\delta|d) = 0$ for all $i \geq 2$, the global symmetries can all be incorporated in the master equation (12). This is the case for the most interesting physical theories, since, for example, $H_i\,(\delta|d) = 0 \; \forall i \geq 2$ for Yang-Mills theories with a semi-simple gauge-group, their supersymmetric extensions and also for gravity [15]. We stress, however, that even in those cases, the existence of the extended formalism based on equation (12) is a nontrivial property which is not automatic and needed demonstration as the above counterexamples indicate.

Finally, we have shown how one can remove the obstructions in all these counterexamples by further extending the formalism. In fact this just illustrates the general case: one can set up an extended antifield formalism, generalizing the one considered here, where obstructions are absent and the higher order conservation laws are encoded in the formalism too [23].

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